1-Apr-2018 EQUIVALENCE RELATION

~~rediffining~~ redefining "equal" i.e "=".

Eg: Let set $B = \mathbb{Z}$, defining "=" on B s.t. $a=b$ if $5 \mid (a-b)$ in $\mathbb{Z}$. (factors have to be Integer).
1) Show that "=" is an equivalence relation on B.
2) Find all equivalence classes of "=".
3) Does $(3,10) \in$ "=" ?
   Does $(7,12) \in$ "=" ?

Ans: 1) We need to show 3 things:
 I) A—A / reflexive (txt book calls it symmetric)
   Means "$a = a$" for every $a \in B$

 ii) A—B / symmetric
   Means if "$a=b$" $\forall a,b \in B$, then "$b=a$".

 iii) A—B—C / transitive
   Means if "$a=b$" and "$b=c$" for some $a,b,c \in B$, then "$a=c$".

A–A: Let $a \in B$. Show "$a=a$" i.e show $5 \mid (a-a)$.
  $a-a=0$, $5 \mid 0$, $0 = 5 \times 0$, $0 \in \mathbb{Z}$.

A–B: Assume "$a=b$" for some $a,b \in B$. Show "$b=a$".
  i.e assume $a-b = 5k_1$, for some $k_1 \in \mathbb{Z}$.
  Multiply by -1: $b-a = 5 \times (-k_1)$, $-k_1 \in \mathbb{Z}$
  Hence $b=a$.

A–B–C: Assume "$a=b$" and "$b=c$" for some $a,b,c \in \mathbb{Z}$. Show "$a=c$".
  $a-b = 5k_1$ and $b-c = 5k_2$ for some $k_1, k_2 \in \mathbb{Z}$.
  Add; $a-b+b-c = 5k_1 + 5k_2$
  $a-c = 5(k_1+k_2)$, $k_1+k_2 \in \mathbb{Z}$.
  Hence $a=c$.

$\therefore$ "=" is an equivalence relation

2) $\bar{0} = [0]$ (set of all numbers that "=" 0.)
 $[0] = \{\dots, -5, 0, 5, 10, 15, \dots\}$. i.e $5n$, $n \in \mathbb{Z}$.
 $[10] = [0]$
 or $[100] = [0]$

 $\bar{1} = [1] = \{\dots, -9, -4, 1, 6, 11, \dots\}$ set of all numbers that "=" 1.
  i.e. $5n+1$, $n \in \mathbb{Z}$

 $[2] = \{\dots, -8, -3, 2, 7, \cancel{13}\ 12, \dots\}$ $5n+2$, $n \in \mathbb{Z}$.

 $[3] = \{\dots, -7, -2, 3, 8, 13, \dots\}$

 $[4] = \{\dots, -6, -1, 4, 9, 14, \dots\}$.

Fact: intersection of any 2 distinct equivalence classes is empty.
 union of all equivalence classes is whole set B.
 Equivalence relation partitions the set to subsets.

~~next~~ 30) We view elements of the new relation as a subset of $B \times B = \{ (a_1, a_2) \mid a_1, a_2 \in B \}$
$(3, 10) \in$ "=" means $3 = 10$.
check: $3 - 10 = -7$
$5 \nmid -7$. Hence, $(3,10) \notin$ "=".

$(7, 12) \in$ "=" means $7 = 12$
Check: $7 - 12 = -5$
$5 \mid -5$. Hence $(7,12) \in$ "=".

Homework 11:

Ques. 1: Let $A = \{0011, 1011, 0101, 0111, 1111, 1101\}$. Define = on A, where if $a, b \in A$ then $a = b$ if number of zero digits on a = no. of zero digits on b.
i) Convince me that "=" is an equivalence relation.
ii) Find all equivalence classes of (A, "=")
iii) View "=" as a subset of $A \times A$. How many elements does "=" have?
iv) Write down all elements of "=".

Ans: i) check:
A—A. let $a \in A$. show "a = a".
Meaning number of zero digits in a = number of zero digits in a. This is true by observation. 0101 = 0101
1011 = 1011.
Axiom 1 holds.

A—B. let $a, b \in A$. If "a = b", then show "b = a".
Note: set A is finite. This means we can prove by example instead of by argument.
Example; let a = 0011 and b = 0101.
"a = b" (0011 = 0101) because number of zero digits in 0011 = no. of zero digits in 0101

No. of zero digits in 0101 (b) = no. of zero digits in 0011 (a)
Hence "b = a". Axiom 2 holds.

A—B—C. let $a, b, c \in A$. If "a = b" and "b = c" show "a = c".
Example: let a = 1011, b = 0111, c = 1101.
1011 = 0111, no. of zero digits in 1011 = no. of zero digits in 0111.
0111 = 1101, no. of zero digits in 0111 = no. of zero digits in 1101.
$\therefore$ no. of zero digits in 1011 = no. of zero digits in 1101.
1011 = 1101
a = c. Axiom 3 holds.

Hence, "=" is an equivalence relation.

ii) [1111] = {1111}
[1011] = {1011, 0111, 1101}
[0011] = {0011, 0101}

iii) No. of elements $= 1 + 3^2 + 2^2$
$= 14$.

iv) (1111, 1111)
(1011, 1011) (1011, 0111) (1011, 1101)
(0111, 1011) (0111, 0111) (0111, 1101)
(1101, 1011) (1101, 0111) (1101, 1101)
(0011, 0011) (0011, 0101) (0101, 0011) (0101, 0101)

Ques 2: Let $A = \{1, 5, 7, 9, 16, 22\}$. Define $=$ on A, where if $a, b \in A$, then $a = b$ if $a \mid b$ (in A). Convince me this is not an equivalence relationship.

Ans: Check:

A – A. let $a \in A$. show "$a = a$".
let $a = 5$. $5 = 5 \times 1$, $1 \in A$. Axiom 1 holds.

A – B. let $a, b \in A$. Assume "$a = b$". Show "$b = a$".
let $a = 1$, $b = 7$. $7 = 7 \times 1$, $7 \in A$.
$1 = 7 \times \frac{1}{7}$, $\frac{1}{7} \notin A$. Axiom 2 fails to hold. ("$b \neq a$").

$\therefore$ "$=$" is not an equivalence relation on A.

Ques 3: Let $A = \{5, 7, 9, 16, 22\}$. Define "$=$" on A where if $a, b \in A$ then $a = b$ if $a \mid b$ (in A). Convince me this is not an equivalence relationship.

Ans: Check:

A – A. let $a \in A$. show "$a = a$".
let $a = 5$. $5 = 5 \times 1$, $1 \notin A$. Axiom 1 fails to hold.

$\therefore$ "$=$" is not an equivalence relation on A.

Ques 4: Let $A = \{5, 7, 9, 11, 19, 20\}$. Define "$=$" on A, where if $a, b \in A$, then $a = b$ if $a \pmod 4 = b \pmod 4$.

i) Convince me that "$=$" is an equivalence relation.
ii) Find all equivalence classes of $(A, =)$.
iii) View "$=$" as a subset of $A \times A$. How many elements does "$=$" have.
iv) ~~Ho~~ Write down the elements of "$=$".

Ans: i) Check:

A – A. Let $a \in A$. Show "$a = a$".
let $a = 5$. $a \bmod 4 = a \bmod 4$
i.e $5 \bmod 4 = 5 \bmod 4$
$(5 - 5) \bmod 4 = 0$
$0 \bmod 4 = 0$. Axiom 1 holds.

A – B. Let $a, b \in A$. Assume "$a = b$".
i.e $a \bmod 4 = b \bmod 4$. let $a = 5$ and $b = 9$
$(a - b) \bmod 4 = 0$
$(5 - 9) \bmod 4 = 0$
$-4 \bmod 4 = 0$
Show "$b = a$".
$\times -1$; $-(a - b) \bmod 4 = 0$
$(b - a) \bmod 4 = (9 - 5) \bmod 4 = 4 \bmod 4 = 0$.
Axiom 2 holds. $(b = a)$

A – B – C. Let $a, b, c \in A$. Assume "$a = b$" and "$b = c$".
i.e $(a - b) \bmod 4 = 0$ and $(b - c) \bmod 4 = 0$
let $a = 7$, $b = 11$, $c = 19$.
$(7 - 11) \bmod 4 = -4 \bmod 4 = 0$, $(11 - 19) \bmod 4 = -8 \bmod 4 = 0$.
Add; $(7 - 11 + 11 - 19) \bmod 4$
$= -12 \bmod 4$
$= 0$.
$\therefore$ $7 = 19$, $a = c$. Axiom 3 holds. Hence, "$=$" is an equivalence relation on A.

ii) $[5] = \{5, 9\}$
$[7] = \{7, 11, 19\}$
$[20] = \{20\}$

iii) No. of elements $= 2^2 + 3^2 + 1$
$= 14$

iv) $(5,5)$ $(5,9)$ $(9,5)$ $(9,9)$
$(7,11)$ $(7,7)$ $(7,19)$
$(11,7)$ $(11,11)$ $(11,19)$
$(19,7)$ $(19,11)$ $(19,19)$
$(20,20)$

Ques 5 Let $A = \mathbb{Z}$. Define $=$ on A, where if $a, b \in A$, then $a = b$ if $7 \mid (a-b)$ (in $\mathbb{Z}$).
i) Convince me this is an equivalence relationship.
ii) Find all equivalence classes of $(A, =)$.
iii) View $=$ as a subset of $A \times A$. Is $(3,10) \in =$? Is $(4,12) \in =$?

Answer: i) check:
A—A. Let $a \in A$. Show "$a = a$".
$a - a = 0 = 7 \times 0$, $0 \in \mathbb{Z}$. Axiom 1 holds.

A—B. Let $a, b \in A$. Assume "$a = b$". Show "$b = a$."
$a - b = 7 \times k_1$, for some $k_1 \in \mathbb{Z}$.
$\times -1$; $b - a = 7 \times (-k_1)$, $-k_1 \in \mathbb{Z}$.
Hence "$b = a$". Axiom 2 holds.

A—B—C. Let $a, b, c \in A$. Assume "$a = b$" and "$b = c$". Show "$a = c$".
$a - b = 7 \times k_1$, $k_1 \in \mathbb{Z}$ $b - c = 7k_2$, $k_2 \in \mathbb{Z}$.
Add; $a - b + b - c = 7k_1 + 7k_2$
$a - c = 7(k_1 + k_2)$, $k_1 + k_2 \in \mathbb{Z}$.
Hence "$a = c$". Axiom 3 holds.

$\therefore$ "$=$" is an equivalence relation on A.

ii) $[0] = \{\ldots, -14, -7, 0, 7, 14, 21, \ldots\}$
$[1] = \{\ldots, -13, -6, 1, 8, 15, 22, \ldots\}$
$[2] = \{\ldots, -12, -5, 2, 9, 16, 23, \ldots\}$
$[3] = \{\ldots, -11, -4, 3, 10, 17, 24, \ldots\}$
$[4] = \{\ldots, -10, -3, 4, 11, 18, 25, \ldots\}$
$[5] = \{\ldots, -9, -2, 5, 12, 19, 26, \ldots\}$
$[6] = \{\ldots, -8, -1, 6, 13, 20, 27, \ldots\}$

iii) $3 - 10 = -7 = 7 \times -1$, $-1 \in \mathbb{Z}$
$\therefore (3, 10) \in =$.

$4 - 12 = -8 = 7 \times -\frac{8}{7}$, $-\frac{8}{7} \notin \mathbb{Z}$

$\therefore (4, 12) \notin =$.

Question 6: Let $A = \{-1, 0, 1, 7, 10, 16, 19\}$. Define "=" on A, where if $a,b \in A$, then $a = b$ if $3 \mid (a-b)$ (in A).
i) Convince me = is an equivalence relation on A.
ii) Find all equivalence classes of $(A, =)$.
iii) View "=" as a subset of $A \times A$. How many elements does = have?
iv) Write down all elements of =.

Answer: i) Note: This is a finite set. Prove by example.

Check:

A-A. This axiom holds because for every element a in A, $a - a = 3 \times 0$, and $0 \in A$.

A-B. $\forall a,b \in A$, if $a = b$, show $b = a$. Let $a = 7$, $b = 10$,
$7 - 10 = -3 = 3 \times -1$, $-1 \in A$.
$x-1$; $10 - 7 = 3 = 3 \times 1$, $1 \in A$.
$\therefore$ "$b = a$". Axiom 2 holds. $16 = 19$ is true as well.

A-B-C. $\forall a,b,c \in A$, if $a = b$ and $b = c$, show $a = c$.
There are no elements in A for the first statement to be true. So by default, the second statement, $a = c$ is true. Axiom 3 holds.

$\therefore$ = is an equivalence relation on A.

ii) $[-1] = \{-1\}$
$[0] = \{0\}$
$[1] = \{1\}$
$[7] = \{7, 10\}$
$[16] = \{16, 19\}$

iii) No. of elements $= 1 + 1 + 1 + 2^2 + 2^2$
$= 11$

iv) $(-1,-1)$ $(0,0)$ $(1,1)$
$(7,10)$ $(7,7)$ $(10,7)$ $(10,10)$
$(16,16)$ $(16,19)$ $(19,16)$ $(19,19)$

Question 7: Let $A = \{-1, 0, 1, 7, 10, 16, 19, 22\}$. Define "=" on A, where if $a,b \in A$ then $a = b$ if $3 \mid (a-b)$ (in A). Convince me this is not an equivalence relationship

Answer: check.
Axioms 1 & 2 hold.

A-B-C. If $a = b$ and $b = c$ for some $a,b,c \in A$. Show $a = c$.
Let $a = 16$, $b = 19$, $c = 22$.
$16 - 19 = -3 = 3 \times -1$, $-1 \in A$.
$19 - 22 = -3 = 3 \times -1$, $-1 \in A$
Add; $16 - 19 + 19 - 22 = 3(-1) + 3(-1)$.
$16 - 22 = 3 \times (-2)$, $-2 \notin A$.
Axiom 3 does not hold. "$a \neq c$".
$\therefore$ = is not an equivalence relationship.

BRUNNEN

Question 8: Convert $(257A)_{11}$ to base 10.

Answer: $2 \times 11^3 + 5 \times 11^2 + 7 \times 11 + 10 = (3354)_{10}.$

Question 9: a) Convert 240016 to base 16.

b) Find $(AF9E1)_{16} - (42326)_{16}$

Answer: a)

$$\begin{array}{r} 15001 \\ 16 \overline{\smash{)}\,240016} \\ -240016 \\ \hline 0 \end{array} \rightarrow \begin{array}{r} 937 \\ 16 \overline{\smash{)}\,15001} \\ -14992 \\ \hline 9 \end{array} \rightarrow \begin{array}{r} 58 \\ 16 \overline{\smash{)}\,937} \\ -928 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 3 \\ 16 \overline{\smash{)}\,58} \\ -48 \\ \hline 10 \end{array} \rightarrow \begin{array}{r} 0 \\ 16 \overline{\smash{)}\,3} \\ -0 \\ \hline 3 \end{array}$$

$(240016)_{10} = (3A990)_{16}$

b)

$$\begin{array}{r} D\ 16 \\ AF9E1 \\ -\ 42326 \\ \hline (6D6BB)_{16} \end{array}$$

5-Apr-2018 PARTIAL ORDER RELATION ON A SET.

redefining "$\leq$".

Partial order on A

δ. A $\to$ set

Definition: We say "$\leq$" is a partial order relation on A (poset) if:

1) A—A (reflexive) $\forall a \in A$, "$a \leq a$"

2) A—B, but not B—A (anti symmetric)
if $a, b \in A$, and $a \neq b$, then if $a \leq b$, $b \nleq a$.

3) A—B—C (transitive)
$\forall a, b, c \in A$, if "$a \leq b$" and "$b \leq c$", then "$a \leq c$".

Example: $A = N^* = \{1, 2, 3, \dots\}$ define "$\leq$" on A. $\forall a, b \in A$, "$a \leq b$" if $b \mid a$ (in A). Show that "$\leq$" is a partial order on A.

Answer: Note: the set A is infinite, so need to prove w/ mathematical argument
Check:
A—A. It is clear that $a \mid a$ (in A) $\forall a \in A$, since $1 \in A$. Hence $a \leq a$.

A—B but not B—A. Assume "$a \leq b$" where ~~b~~ $a \neq b$. We show that "$b \nleq a$".
We have $b \mid a$ for some $a, b \in A$. and "$a \neq b$". We know $a \nmid b$. Thus "$b \nleq a$".

Assuming $a \leq b$, means $a = kb$ for some k in N^*
$\div k$; $\frac{1}{k} a = b$, but $\frac{1}{k} \notin N^*$
Hence $a \mid b$, so $b \nleq a$.

A—B—C. Assume "$a \leq b$" and "$b \leq c$".
So $b \mid a$ and $c \mid b$ for some $a, b, c \in A$.
$a = bn_1$ and $b = cn_2$, for some $n_1, n_2 \in A$
$a = cn_2 n_1$, $n_2 n_1 \in A$.
$\therefore$ $c \mid a$, thus "$a \leq c$".

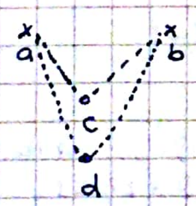
Hence, "$\leq$" is a partial order relation on A.

Fact: If "$\leq$" is a partial order, we donot have equivalence classes (because Axiom 2 fails).

Is $(A, \leq)$ a lattice? (use Hasse diagram)?
Ans: (a relation that is not partial order cannot be lattice. the question becomes meaningless).

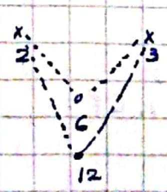
$\forall a, b \in A$, $a \wedge b$ and $a \vee b$ must exist.
$a \wedge b \to$ greatest lower bound of a, b
$a \vee b \to$ least upper bound of a, b.

Greatest lower bound of a, b



can we find c s.t $c \leq b$ and ~~b~~ $c \leq a$
and d s.t $d \leq b$ and $d \leq a$ and $d \leq c$.
If yes, $a \wedge b$ exists. If $d \nleq c$ then $a \wedge b$ does not exist.

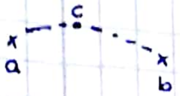
eg: $2 \wedge 3$.



$6 \leq 3$ and $6 \leq 2$.
$12 \leq 3$ and $12 \leq 2$ and $12 \leq 6$.

For this example, $a \wedge b$ is LCM $[a, b]$

least upper bound of a, b.



$a \leq c$ and $b \leq c$

for this example, $a \vee b = \gcd(a, b)$

So "$\leq$" is a lattice.

Find $6 \wedge 14$

<u>Ans:</u> $6 \wedge 14 = \cancel{LM}$ LCM $[6, 14] = \dfrac{6 \times 14}{\gcd(6,14)} = \dfrac{6 \times 14}{6 \vee 14} = \dfrac{6 \times 14}{2}$

$$\begin{array}{r|l} 2 & 6, 14 \\ 3 & 3, 7 \\ 7 & 1, 7 \\ & 1, 1 \end{array}$$

$2 \times 3 \times 7 = 42.$

Hence $6 \wedge 14 = 42$ means $42 \leq 14$ and $42 \leq 6$

$14 \mid 42$ $\quad$ $6 \mid 42$

$6 \vee 14 = 2$ means $6 \leq 2$ and $14 \leq 2$

$2 \mid 6$ $\quad$ $2 \mid 14$

Find $3 \wedge 9$ and $3 \vee 9$

<u>Ans</u> $3 \wedge 9 = 9$, $3 \vee 9 = 3$.

<u>Example:</u> Same example as above but $A = \mathbb{Z}^*$.

"$\leq$" is not a poset on A. Why? Axiom 2 fails.

eg let $a = 2$ & $b = -2$

"$a \leq b$" because $-2 \mid 2$ in $\mathbb{Z}$.

but "$b \leq a$" because $2 \mid -2$ in $\mathbb{Z}$.

So A – B becomes symmetric in some cases.

<u>Question:</u> let $A = \{2, 4, 8, 10, 100\}$.

Define "$a \leq b$" $\forall a, b \in A$ means $a \mid b$ in A. Is "$\leq$" a poset on A?

<u>Ans:</u> A – A fails since $2 \nmid 2$ in A

$2 = 2 \times 1$ and $1 \notin A$.

<u>Question:</u> let $A = \{1, 2\}$

$|P(A)| = 2^{|A|} = 2^2 = 4$

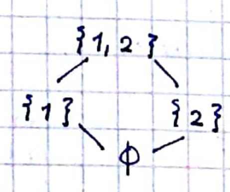
$P(A) = \{\{1,2\}, \phi, \{1\}, \{2\}\}$.

Define "$\leq$" on $P(A)$. $\forall a, b \in P(A)$ "$a \leq b$" means $a \subseteq b$. You may check that "$\subseteq$" is a partial order.

Is "$A, \leq$" a lattice?

<u>Ans:</u> Note that $P(A)$ is finite.

Hasse diagram:



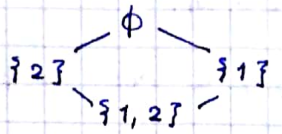
$\phi \subseteq \{1\}$ and $\phi \subseteq \{2\}$, $a \wedge b$ exists.

$\{1\} \subseteq \{1, 2\}$ and $\{2\} \subseteq \{1, 2\}$.

$\{2\} \wedge \{1, 2\} = \{2\}$

$\{2\} \vee \{1, 2\} = \{1, 2\}$

Question: Same as above except "$a \leq b$" if $b \subseteq a$.
Hasse diagram:



$\{1\} \vee \{2\} = \phi$
$\{1\} \wedge \{2\} = \{1,2\}. = A.$

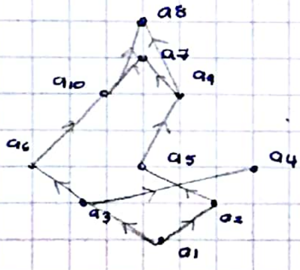
10-Apr-2018 **Question:** Let $A = \{1,2\}$
$(P(A), \leq)$, $\forall a,b \in P(A)$ "$a \leq b$" means $a-b \in \{\phi, \{1\}\}$
Show this is not poset.
Ans: $A-A$, Let $a \in P(A)$
~~a-l~~ $a - a = \phi \in \{\phi, \{1\}\}$. Axiom 1 holds.

$P(A) = \{\{1,2\}, \phi, \{1\}, \{2\}\}$.

$A-B$ but not $B-A$. Let $a,b \in P(A)$.
Let $a = \{1\}$, $b = \{2\}$.
$a \leq b$ means $a-b = \{1\} \in P(A)$.
$b-a = \{2\} \notin P(A)$ so $b \nleq a$.
Now let $a = \phi$, $b = \{1\}$.
$a \leq b$ means $a-b = \phi \in P(A)$.
$b-a = \{1\} \in P(A)$ meaning $b \leq a$. So, axiom 2 fails.

$\therefore$ "$\leq$" is not poset on $P(A)$.

Question: Given the Hasse diagram of a partial order relation on a set B
Is this diagram a lattice?



check:
$a_1 \wedge a_2 = a_1$
$a_1 \vee a_2 = a_2$

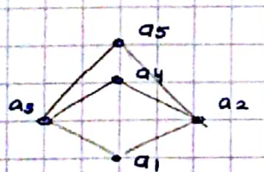
$a_1 \leq a_2$

$a_1 \wedge a_4 = a_1$ $\quad a_1 \leq a_3 \leq a_4 \therefore a_1 \leq a_4$ (transitive)
$a_1 \vee a_4 = a_4$

$a_4 \wedge a_5 = a_1$ $\quad a_1 \leq a_3 \leq a_4 \therefore a_1 \leq a_4$
$a_1 \leq a_2 \leq a_5 \therefore a_1 \leq a_5$
$a_4 \vee a_5 =$ does not exist

$\therefore$ Not a lattice.
Note: In Hasse diagram we donot draw transitive relation, it is assumed. Eg, no line betw. a_1 and a_4 but $a_1 \leq a_4$ because $a_1 \leq a_3 \leq a_4$.

Question:



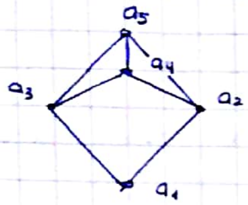
Show this is not lattice.
check:
$a_1 \wedge a_2 = a_1$
$a_1 \vee a_2 = a_2$

$a_1 \wedge a_5 = a_1$
$a_1 \vee a_5 = a_5$

$a_4 \wedge a_5 =$ does not exist. why?
$a_2 \leq a_4$ and $a_2 \leq a_5$
but $a_3 \leq a_4$ and $a_3 \leq a_5$ and $a_2 \nleq a_3$ nor $a_3 \nleq a_2$
$a_4 \vee a_5 =$ does not exist. $\therefore$ This is not a lattice

$a_4 \wedge a_5 = a_1$ is wrong. Why?
because $a_2 \leq a_4$ and $a_2 \leq a_5$ but $a_2 \nleq a_1$.
However $a_1 \leq a_2$.
So can $a_4 \wedge a_5 = a_2$? No, because $a_3 \nleq a_2$ nor $a_2 \nleq a_3$.
or $a_4 \wedge a_5 = a_3$?

Question:



is a lattice.

Note: $a \wedge b = c$ means $c \leq a$ and $c \leq b$
if $m \leq a$ and $m \leq b$, then $m \leq c$. If not, then $a \wedge b \neq c$

$a \vee b = c$ means $a \leq c$ and $b \leq c$
if $a \leq m$ and $b \leq m$, then $c \leq m$. If not, then $a \vee b \neq c$.

Note. when ~~will~~ writing equivalence classes of infinite eg $\dot{a} = b$ if $6 \mid (a-b)$
eg $[0] = \{\ldots, 0, 6, \ldots\}$
can also be written as: $[0] = 0 + 6\mathbb{Z}$ $\{6a \mid a \in \mathbb{Z}\}$

12-Apr-2018

Definition: assume $(A, \leq)$ is a poset.
an element $m \in A$ is called a maximum element if $a \leq m$, $\forall a \in A$.
an element $d \in A$ is called a minimum element if $d \leq a$, $\forall a \in A$.

We can view "$\leq$" as a subset of $A \times A$.
$(a, b) \in \leq$ means $a \leq b$
and $b \nleq a$, $\Rightarrow$ $(b, a) \notin \leq$